

Spin relaxation time, spin dephasing time and ensemble spin dephasing time in *n*-type GaAs quantum wells

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We investigate the spin relaxation and spin dephasing of *n*-type GaAs quantum wells. We obtain the spin relaxation time T_1 , the spin dephasing time T_2 and the ensemble spin dephasing time T_2^* by solving the full microscopic kinetic spin Bloch equations, and we show that, analogous to the common sense in an isotropic system for conduction electrons, T_1 , T_2 and T_2^* are identical due to the short correlation time. The inhomogeneous broadening induced by the D'yakonov-Perel term is suppressed by the scattering, especially the Coulomb scattering, in this system.

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Much attention has been devoted to the spin degree of freedom of carriers in zinc-blende semiconductors, both in bulk systems and in reduced dimensionality structures, like quantum wells and quantum dots. Understanding spin dephasing and spin relaxation of carriers in these systems is a key factor for the realization of high quality spintronic devices.^{1,2,3,4} Of special interest is the calculation of quantities known as spin relaxation time, T_1 , and spin dephasing time, T_2 . T_1 is defined as the time it takes for the spins along the longitudinal field to reach equilibrium. Therefore, it is related with the relaxation of the average spin polarization. On the other hand, T_2 is defined as the time it takes for the transverse spins, initially precessing in phase about the longitudinal field, to lose their phase.⁴ In general $T_2 \leq 2T_1$, and $T_1 = T_2$ is believed to be true when the system is isotropic and the correlation time for the interaction is very short compared with the Larmor period.^{5,6}

A qualitative reason for $T_1 = T_2$ is that if the correlation time is short compared with the Larmor period the interaction with the magnetic fields is not affected by a transformation into a coordinate system rotating at the Larmor frequency. The surrounding seems isotropic and the rate of decay will be the same for all directions. Therefore, longitudinal and transverse relaxation times will be the same. Hence the decay of the spin signal will be the same in all directions and T_1 equals T_2 , as argued in Ref. 5. For several years T_1 and T_2 were considered as the only important factors describing the spin dynamics under external fields.

In recent years, however, many experiments have been performed reflecting the dephasing process of the ensemble of electrons, instead of the dynamics of a single one.⁷ In fact, electrons with different momentum states have different precession frequencies due to the momentum dependence of the effective magnetic field acting on the electron spin, and this inhomogeneity of precession frequencies can cause a reversible phase loss. A parameter name, T_2^* , was coined to describe the dephasing process associated to this inhomogeneous broadening of the pre-

cessing frequencies.

Wu *et al.* have already shown that in the presence of this inhomogeneous broadening, any scattering, including the spin-conserving scattering, can cause irreversible spin dephasing.^{8,9,11} This fact leads to the belief that, in general, $T_2^* \leq T_2$. However, for conduction electrons $T_2^* = T_2$ is known to be a very good approximation because the inhomogeneous broadening is always inhibited by the relatively strong scattering existing in the system.¹³

In this paper, we investigate the spin relaxation and dephasing of electrons in *n*-type GaAs quantum wells (QWs) grown in the (100) direction, considered to be the z axis. The width of the well, a , is assumed to be small enough for having just the lowest subband occupied. A moderate magnetic field \mathbf{B} is applied along the x axis (in the Voigt configuration).

We calculate T_1 , T_2 and T_2^* of the electron by numerically solving the kinetic spin Bloch equations including scattering by phonons and impurities, besides of the Coulomb scattering due to electron-electron interaction. Then we show that T_1 , T_2 and T_2^* are identical in these QWs, as the scattering here is relatively strong.

In the present full microscopic treatment we associate the above parameters with the decay slope of the envelope of $\rho_{\mathbf{k},\sigma\sigma'}$, the single-particle density matrix elements:

(i) T_1 is determined from the slope of the envelope of

$$\Delta N = \sum_{\mathbf{k}} (f_{\mathbf{k},\uparrow} - f_{\mathbf{k},\downarrow}) ; \quad (1)$$

(ii) T_2 is associated with the incoherently summed spin coherence¹²

$$\rho = \sum_{\mathbf{k}} |\rho_{\mathbf{k}}(t)| ; \quad (2)$$

(iii) Finally, T_2^* is defined from the slope of the envelope of the coherently summed spin coherence

$$\rho' = \left| \sum_{\mathbf{k}} \rho_{\mathbf{k}}(t) \right| . \quad (3)$$

In these equations $\rho_{\mathbf{k},\sigma\sigma} \equiv f_{\mathbf{k},\sigma}$ describes the electron distribution functions of wave vector \mathbf{k} and spin σ . The off-diagonal elements $\rho_{\mathbf{k},\uparrow\downarrow} = \rho_{\mathbf{k},\downarrow\uparrow}^* \equiv \rho_{\mathbf{k}}$ describe the inter-spin-band correlations for the spin coherence.

With the DP term¹⁴ included, the Hamiltonian of the electrons can be written as:

$$H = \sum_{\mathbf{k}\sigma\sigma'} \{ \varepsilon_{\mathbf{k}} \delta_{\sigma\sigma'} + [g\mu_B \mathbf{B} + \mathbf{h}(\mathbf{k})] \cdot \frac{\boldsymbol{\sigma}_{\sigma\sigma'}}{2} \} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma'} + H_I . \quad (4)$$

Here $\varepsilon_{\mathbf{k}} = \mathbf{k}^2/2m^*$ is the energy of the electron with wave vector \mathbf{k} . $\boldsymbol{\sigma}$ represents the Pauli matrices. For wide-band-gap semiconductors such as GaAs, unless a very large bias voltage is applied,¹⁵ the DP term has its major contribution coming out of the Dresselhaus term.¹⁶ Then, we have:

$$\begin{aligned} h_x(\mathbf{k}) &= \gamma k_x(k_y^2 - \langle k_z^2 \rangle) , \\ h_y(\mathbf{k}) &= \gamma k_y(\langle k_z^2 \rangle - k_x^2) , \\ h_z(\mathbf{k}) &= 0 . \end{aligned} \quad (5)$$

Here $\gamma = (4/3)(m^*/m_{cv})(1/\sqrt{2m^*E_g})(\eta/\sqrt{1-\eta/3})$,

$\eta = \Delta/(E_g + \Delta)$ in which E_g denotes the band gap, Δ represents the spin-orbit splitting of the valence band, m^* stands for the electron mass in GaAs, and m_{cv} is a constant close in magnitude to free electron mass m_0 . In the infinite-well-depth approximation, $\langle k_z^2 \rangle$ is $(\pi/a)^2$. The interaction Hamiltonian H_I in Eq. (4) is composed of the electron-electron Coulomb interaction, electron-AC-phonon scattering and electron-LO-phonon scattering, as well as electron-impurity scattering. Their expressions can be found in textbooks.¹⁷

We construct the many-body kinetic spin Bloch equations by the non-equilibrium Green function method¹⁸ as follows:

$$\dot{\rho}_{\mathbf{k},\sigma\sigma'} = \dot{\rho}_{\mathbf{k},\sigma\sigma'}|_{coh} + \dot{\rho}_{\mathbf{k},\sigma\sigma'}|_{scatt} . \quad (6)$$

Here $\dot{\rho}_{\mathbf{k},\sigma\sigma'}|_{coh}$ describes the coherent spin precessions around the applied magnetic field \mathbf{B} in the Voigt configuration, the effective magnetic field $\mathbf{h}(\mathbf{k})$, and the effective magnetic field from the electron-electron Coulomb interaction in the Hartree-Fock approximation. This coherent part can be written as:

$$\frac{\partial f_{\mathbf{k},\sigma}}{\partial t} \Big|_{coh} = -2\sigma \{ [g\mu_B B + h_x(\mathbf{k})] \text{Im} \rho_{\mathbf{k}} + h_y(\mathbf{k}) \text{Re} \rho_{\mathbf{k}} \} + 4\sigma \text{Im} \sum_{\mathbf{q}} V_{\mathbf{q}} \rho_{\mathbf{k}+\mathbf{q}}^* \rho_{\mathbf{k}} , \quad (7)$$

$$\frac{\partial \rho_{\mathbf{k}}}{\partial t} \Big|_{coh} = \frac{1}{2} [ig\mu_B B + ih_x(\mathbf{k}) + h_y(\mathbf{k})] (f_{\mathbf{k},\uparrow} - f_{\mathbf{k},\downarrow}) + i \sum_{\mathbf{q}} V_{\mathbf{q}} [(f_{\mathbf{k}+\mathbf{q},\uparrow} - f_{\mathbf{k},\downarrow}) \rho_{\mathbf{k}} - \rho_{\mathbf{k}+\mathbf{q}} (f_{\mathbf{k},\uparrow} - f_{\mathbf{k},\downarrow})] , \quad (8)$$

in which $V_{\mathbf{q}}$ denotes the Coulomb potential and its expression can be found in Ref. 10. In Eq. (6) $\dot{\rho}_{\mathbf{k},\sigma\sigma'}|_{scatt}$ denotes the Coulomb electron-electron, electron-phonon and electron-impurity scattering. The expressions for these scattering terms and the details of solving these many-body kinetic spin Bloch equations are laid out in detail in Ref. 10.

We numerically solve the kinetic spin Bloch equations and obtain temporal evolution of the electron distribution $f_{\mathbf{k},\sigma}(t)$ and the spin coherence $\rho_{\mathbf{k}}(t)$. The material parameters of GaAs in our calculation are the same with the parameters in Ref. 10. In our calculations the width of the QW is chosen to be 15 nm; the initial spin polarization $P_\lambda = \Delta N/N$ is 2.5 % and the magnetic field $B = 4$ T.

In Fig. 1 we show the typical evolution of ρ , ρ' and ΔN for $T = 120$ K, the total electron density $N = 4 \times 10^{11}$ cm⁻² and the impurity density $N_i = 0$. It is seen from the figure that ρ , ρ' and ΔN are all oscillating due to the presence of the magnetic field in Voigt configuration. From the envelope of ρ and ΔN , we see that $T_1 = T_2$. This result can be understood as a consequence of the momentum relaxation time here being less than 1 pi-

cosecond. This is several orders of magnitude smaller than the period of the effective Larmor precession induced by the DP term $[2\pi/|\mathbf{h}(\mathbf{k})|]|_{k=k_f} = 26$ ps, as well as the Larmor period of magnetic field $2\pi/\omega_B = 40$ ps. Therefore, in the condition of impurity-free n -type GaAs quantum wells, the system is visibly isotropic in the x - y -plane and the rate of decay of spin signal has the same speed in all directions since the correlation time is short compared with the Larmor period.

We can also observe from Fig. 1 that the incoherently summed spin coherence and the coherently summed spin coherence (solid and chained curves) are almost identical, which means $T_2 = T_2^*$. This result indicates that the inhomogeneous broadening induced by the DP term is totally suppressed by the strong scattering coming out of the electron-electron and electron-phonon interactions. To reveal this effect, we investigate the time evolution of electrons in different momentum states, which have different precession frequencies in the presence of the DP term. This inhomogeneity of precession frequencies can cause a reversible phase loss making the coherently summed spin coherence ρ' to decay faster than the incoherently summed spin coherence ρ . However, this ef-

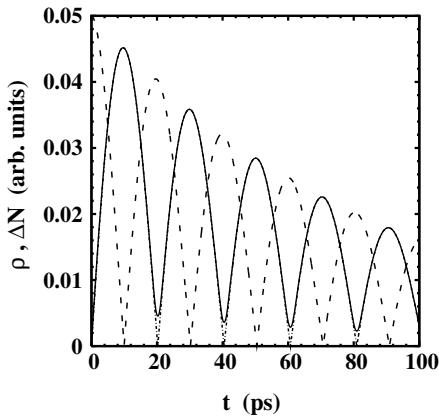


FIG. 1: Typical time evolution of spin density ΔN (dotted curve), the incoherently summed spin coherence ρ (solid curve) and coherently summed spin coherence ρ' (chained curve) for the case of $T = 120$ K, $B = 4$ T, $N = 4 \times 10^{11}$ cm $^{-2}$ and $N_i = 0$.

fect can be inhibited by the strong scattering. To eliminate the effect of the inhibition by strong scattering, we study the case of $T = 120$ K, $N = 4 \times 10^{11}$ cm $^{-2}$ and $N_i = 0$ with no Coulomb electron-electron scattering included and plot the oscillating period of each \mathbf{k} state in Fig. 2. In this case, the total scattering is relatively weak, coming out exclusively from the scattering by phonons, and we can see that electrons with different momentum states do have different oscillating periods although the electron-LO-phonon scattering makes the oscillating period changing with the period of one LO phonon frequency as the difference of the diameters of the nearest two concentric circles differs exactly by one LO phonon frequency. However, even in this case, the contribution of the inhomogeneous broadening is very weak. As discussed in Ref. 19 the inhomogeneity of precession frequencies makes a contribution of $2/\delta\omega_I$ to the total spin dephasing time if the inhomogeneous lineshape is assumed to be Gaussian. Here $\delta\omega_I$ represents the root of the mean square of the precession frequencies and can be written as: $\delta\omega_I = \sqrt{\frac{\sum_{\mathbf{k}}(\omega_{\mathbf{k}} - \bar{\omega})^2 f_{\mathbf{k}}}{\sum_{\mathbf{k}} f_{\mathbf{k}}}}$, with $\bar{\omega} = \frac{\sum_{\mathbf{k}} \omega_{\mathbf{k}} f_{\mathbf{k}}}{\sum_{\mathbf{k}} f_{\mathbf{k}}}$ and $f_{\mathbf{k}}$ representing the Fermi distribution. In this case the $2/\delta\omega_I$ we calculated is 308 ps, while the total T_2 is only 35 ps. Therefore, the contribution of the inhomogeneity can be omitted and the difference between T_2 and T_2^* is still very small.

Furthermore, when we include the Coulomb scattering in this system, we find that electrons in each momentum state has the *same* oscillating period of 40 ps, which is exactly equals to the Larmor period induced by the magnetic field and also the oscillating period of ρ and ρ' . The inhomogeneous broadening is suppressed and T_2 equals

T_2^* .

We further check this result with different temperatures, electron densities and impurity densities. We find that $T_1 = T_2 = T_2^*$ is valid in a very wide range of tem-

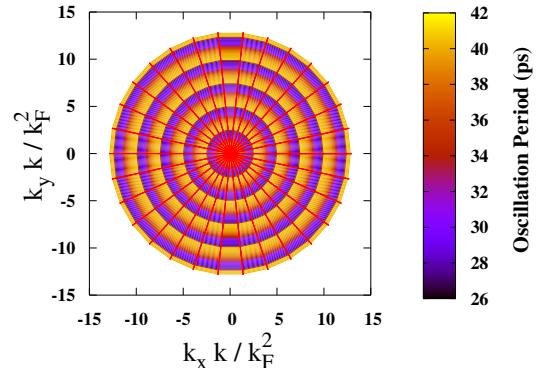


FIG. 2: (Color online) The contour plot of oscillating period vs $k_x k / k_F^2$ and $k_y k / k_F^2$ with k_F representing the Fermi wave vector and $k = |\mathbf{k}|$ for the case of $T = 120$ K, $B = 4$ T, $N = 4 \times 10^{11}$ cm $^{-2}$ and $N_i = 0$. There is no electron-electron Coulomb scattering in the calculation.

perature from 10 K to 300 K, and electron densities from 2×10^{10} to 4×10^{11} cm $^{-2}$. Including the impurity scattering will not change this result. Even in the case of $T = 10$ K, with the total electron density $N = 2 \times 10^{10}$ cm $^{-2}$ and without the impurity and the Coulomb scattering, where the electron-AC-phonon scattering is the dominant scattering, the difference obtained between T_1 , T_2 and T_2^* is still less than 6 %.

In conclusion, we have investigated the spin relaxation and the spin dephasing of electrons in n -type GaAs quantum wells and calculate T_1 , T_2 and T_2^* by numerically solving the kinetic spin Bloch equations. We have obtained that they have the same value in a very wide range of temperatures, electron densities and the impurity densities and we have shown that this behavior is due to the short correlation time. More experiments such as the spin echo experiment²⁰ are needed to check the findings here.

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- ¹ S. A. Wolf, J. Supercond. **13**, 195 (2000).
 - ² *Spin Electronics*, edited by M. Ziese and M. J. Thornton (Springer, Berlin, 2001).
 - ³ *Semiconductor Spintronics and Quantum Computation*, ed. by D. D. Awschalom, D. Loss, and N. Samarth (Springer-Verlag, Berlin, 2002).
 - ⁴ I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
 - ⁵ D. Pines and C. P. Slichter, Phys. Rev. **100**, 1014 (1955).
 - ⁶ R. K. Wangsness and F. Bloch, Phys. Rev. **89**, 728 (1953).
 - ⁷ J. M. Kikkawa, I. P. Smorchkova, N. Samarth, and D. D. Awschalom, Science **277**, 1284 (1997); J. M. Kikkawa and D. D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998); Science **281**, 656 (2000); Science **287**, 473 (2000); G. Salis, D. T. Fuchs, J. M. Kikkawa, D. D. Awschalom, Y. Ohno and H. Ohno, Phys. Rev. Lett. **86**, 2677 (2001); Y. Kato, R. C. Myers, D. C. Driscoll, A. C. Gossard, J. Levy, and D. D. Awschalom, Science **299**, 1201 (2003); H. Hoffmann, G. V. Astakhov, T. Kiessling, W. Ossau, G. Karczewski, T. Wojtowicz, J. Kossut, and L. W. Molenkamp, Phys. Rev. B **74**, 073407 (2006).
 - ⁸ M. W. Wu and C. Z. Ning, Eur. Phys. J. B **18**, 373 (2000); M. W. Wu, J. Phys. Soc. Jpn. **70**, 2195 (2001); J. L. Cheng, M. Q. Weng, and M. W. Wu, Solid State Commun. **128**, 365 (2003).
 - ⁹ M. Q. Weng and M. W. Wu, Phys. Rev. B **68**, 075312 (2003); **71**, 199902(E) (2005); Chin. Phys. Lett. **22**, 671 (2005); M. Q. Weng and M. W. Wu, Phys. Rev. B **70**, 195318 (2004); L. Jiang and M. W. Wu, *ibid.* **72**, 033311 (2005); C. Lü, J. L. Cheng, and M. W. Wu, *ibid.* **73**, 125314 (2006).
 - ¹⁰ M. Q. Weng, M. W. Wu, and L. Jiang, Phys. Rev. B **69**, 245320 (2004); J. Zhou, J. L. Cheng, and M. W. Wu, *ibid.* **75**, 045305 (2007).
 - ¹¹ M. Q. Weng and M. W. Wu, J. Appl. Phys. **93**, 410 (2003); M. Q. Weng, M. W. Wu, and Q. W. Shi, Phys. Rev. B **69**, 125310 (2004); L. Jiang, M. Q. Weng, M. W. Wu, and J. L. Cheng, J. Appl. Phys. **98**, 113702 (2005).
 - ¹² M. W. Wu and H. Metiu, Phys. Rev. B **61**, 2945 (2000).
 - ¹³ R. Dupree and B. W. Holland, Phys. Stat. Sol. (b) **24**, 275 (1967).
 - ¹⁴ M. I. D'yakonov and V. I. Perel', Zh. Eksp. Teor. Fiz. **60**, 1954 (1971) [Sov. Phys. JETP **38**, 1053 (1971)].
 - ¹⁵ W. H. Lau and M. E. Flatté, Phys. Rev. B **72**, 161311(R) (2005).
 - ¹⁶ G. Dresselhaus, Phys. Rev. **100**, 580 (1955).
 - ¹⁷ G. D. Mahan, *Many-particle Physics* (Plenum, New York, 1981).
 - ¹⁸ H. Haug and A. P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer-Verlag, Berlin, 1996).
 - ¹⁹ L. Allen and J. H. Eberly, *Optical Resonance and Two-level Atoms* (Dover, New York, 1975).
 - ²⁰ A. M. Tyryshkin, S. A. Lyon, W. Jantsch, and F. Schäffler, Phys. Rev. Lett. **94**, 126802 (2005).